13

DISSERTATIO ACADEMICA

THEORIAM ÆQUATIONUM FUNCTIONALIUM DUARUM VARIABILIUM EJUSQUE IN DOCTRINA SERIERUM USUM EXHIBENS;

QUAM

CONSENSU AMPLISS. FACULTATIS PHILOSOPH. AD IMPERIALEM ACAD. ABOËNSEM,

PRÆSIDE

Mag. NATH. G. AF SCHULTÉN,

Mathematum Professore Publ. & Ord.,
Acad. Imperialis Scientiarum Petropolitanæ Socio Corresp.,

PRO GRADU PHILOSOPHICO

P. P.

FREDERICUS ÆJMELÉ,
Ostrobotniensis.

In Audit. Jurid. die XXIII Maji MDCCCXXVII.
horis p. m. solitis.

P. IV.

ABOÆ, Typis FRENCKELLIANIS.

THESES.

I.

Quamvis mathematicæ in universum scientiæ inter alias omnes certitudine et evidentia emineant adeo singularibus, ut vix ullum dubitandi locum relinquere videantur, — quamobrem etiam a quovis veritatis amante non possunt non maximi æstimari, — attamen, ultimum si respicias earum principium, a philosophia petendum hoc esse, contendi jure potest.

II.

Quæ inter cognitionem philosophicam et mathematicam a quibusdam philosophis assumta est differentia, quod illa scilicet solis conceptibus, hæc autem *constructione* nititur conceptuum, probanda nobis non videtur.

III.

Evidentia Matheseos non quidem ex *empirico*, sed e *puro* vel *ideali*, intuitu derivanda est.

IV.

Denominationes quædam in Mathesi obtinentes nobis judicibus vitiosæ sunt, quas inter v. gr. Logarithmorum *Hyperbolicorum* Serierumque *Recurrentium* citasse liceat.

assumtis pro p_x duabus functionibus particularibus in allatis supra æquationibus c) et d) obveni-

D entibus

$$\left.\begin{aligned} &+ h'\left((q-1)x^{q-2} + \frac{(q-1)(q-2)}{1.2}x^{q-3}\right.\\ &\left.+ \&c.\right) + h_{,}'\left((q-2)\, x^{q-3}\right.\\ &\left.+ \frac{(q-2)(q-3)}{1.2}x^{q-4} + \&c.\right) + \&c.\\ &+ \&c. \end{aligned}\right\}$$

$$\begin{aligned} p_{x+2} - 2p_{x+1} + p_x &= \left.\begin{aligned} &g''x^{p-2} + g_{,}''x^{p-3} + g_{,,}''x^{p-4} + \&c.\\ &+ h''x^{q-2} + h_{,}''x^{q-3} + h_{,,}''x^{q-4} + \&c.\\ &+ \&c. \end{aligned}\right\}\\ &= p''_x; \end{aligned}$$

hincque, eamdem ob caussam,

$$\begin{aligned} p_{x+3} - 3p_{x+2} + 3p_{x+1} - p_x &= p''_{x+1} - p''_x\\ &= \left.\begin{aligned} &g'''x^{p-3} + g_{,}'''x^{p-4} + g_{,,}'''x^{p-5}\\ &+ \&c.\\ &+ h'''x^{q-3} + h_{,}'''x^{q-4} + h_{,,}'''x^{q-5}\\ &+ \&c.\\ &+ \&c. \end{aligned}\right\}, \end{aligned}$$

entibus, ipsis scilicet

$$\frac{1}{x\,(x+1)\,(x+2)\,\ldots\,(x+m)} \quad \text{et} \quad a^x \operatorname{Sin}(b+cx).$$

Quod

$$\left.\begin{aligned} p_{x+4}-4p_{x+3}+6p_{x+2}-4p_{x+1}+p_x = {} & g''''x^{p-4}+g_{,}''''x^{p-5}+\&c. \\ & +h''''x^{q-4}+h_{,}''''x^{q-5}+\&c. \\ & +\&c. \end{aligned}\right\},$$

sicque porro; sive in genere

$$\left.\begin{aligned} & p_{x+n}-np_{x+n-1}+\frac{n(n-1)}{1\,.\,2}p_{x+n-2} \\ & -\&c. \pm np_{x+1} \mp p_x = g^{(n)}x^{p-n}+g_{,}^{(n)}x^{p-n-1}+\&c. \\ & \qquad\qquad\qquad\quad +h^{(n)}x^{q-n}+h^{(n)}x^{q-n-1}+\&c. \\ & \qquad\qquad\qquad\quad +\&c. \end{aligned}\right\};$$

ex qua quidem æquatione perspicitur, in serie quaque horizontali membri ejus posterioris evanescere tandem necessario debere ipsam x, auctoque dein adhuc unitate numero n, evanescere omnino totam de qua agitur seriem, proque majoribus ipsius n valoribus in formula ulterius non occurrere: cnm ex ipsa scilicet pateat ratione qua istæ successive formantur series, negativos fieri non posse exponentes ipsius x. Sic v. gr. quando $n =$ exponenti p, evanescet in serie prima valoris ultimi generalis

$$g^{(n)}x^{p-n} + g_{,}^{(n)}x^{p-n-1} + \&c.$$

ipsa x, proque $n = p+1$, majoribusque adhuc valori-

Quod ad priorem scilicet earum, habebimus

$$\Sigma \frac{1}{x(x+1)(x+2)\ldots(x+m)} = -\frac{1}{m} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-1)},$$

$$\Sigma^2 \frac{1}{x(x+1)(x+2)\ldots(x+m)} = \Sigma\left(-\frac{1}{m} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-1)}\right)$$

$$= -\frac{1}{m} \cdot \Sigma \frac{1}{x(x+1)(x+2)\ldots(x+m-1)}$$

$$= -\frac{1}{m} \cdot -\frac{1}{m-1} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-2)}$$

$$= +\frac{1}{m(m-1)} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-2)},$$

$$\Sigma^3 \frac{1}{x(x+1)(x+2)\ldots(x+m)} = \Sigma\left(\frac{1}{m(m-1)} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-2)}\right)$$

$$= \frac{1}{m(m-1)} \cdot \Sigma \frac{1}{x(x+1)(x+2)\ldots(x+m-2)}$$

=

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bus, tota evanescet series; sicque porro. Posito igitur in functione data p_x, i. e.

$$gx^p + hx^q + \&c.,$$

maximo exponentium p, q, &c. $= s$, perspicuum utique est, haberi necessario

$$p_{x+n} - np_{x+n-1} + \frac{n(n-1)}{1.2} p_{x+n-2} - \&c. \pm np_{x+1} \mp p_x = o,$$

quando fuerit $n = s + 1$ atque $> s + 1$.

$$= \frac{1}{m(m-1)} \cdot -\frac{1}{m-2} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-3)}$$

$$= -\frac{1}{m(m-1)(m-2)} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-3)}$$

$$\Sigma^4 \frac{1}{x(x+1)(x+2)\ldots(x+m)} = \Sigma\left(-\frac{1}{m(m-1)(m-2)} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-3)}\right)$$

$$= -\frac{1}{m(m-1)(m-2)} \cdot \Sigma \frac{1}{x(x+1)(x+2)\ldots(x+m-3)}$$

$$= -\frac{1}{m(m-1)(m-2)} \cdot -\frac{1}{m-3} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-4)}$$

$$= +\frac{1}{m(m-1)(m-2)(m-3)} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-4)},$$

sicque porro; hincque, per inductionem, in genere

$$\Sigma^n \frac{1}{x(x+1)(x+2)\ldots(x+m)} = \pm \frac{1}{m(m-1)(m-2)\ldots(m-n+1)} \cdot \frac{1}{x(x+1)(x+2)\ldots(x+m-n)},$$

adhibito signo + quando n numerus est *par*, — vero, si idem est *impar*.

Posteriorem vero quod attinet functionem, sive

$$a^x \operatorname{Sin}(b+cx),$$

fiet

fiet

$$\Sigma a^x \operatorname{Sin}(b+cx) = \frac{a^x (a \operatorname{Sin}(b-c+cx) - \operatorname{Sin}(b+cx))}{a^2 - 2a \operatorname{Cos} c + 1}$$

$$\Sigma^2 a^x \operatorname{Sin}(b+cx) = \Sigma \frac{a^x (a \operatorname{Sin}(b-c+cx) - \operatorname{Sin}(b+cx))}{a^2 - 2a \operatorname{Cos} c + 1}$$

$$= \frac{a}{a^2-2a \operatorname{Cos} c+1} \Sigma a^x \operatorname{Sin}(b-c+cx) - \frac{1}{a^2 - 2a \operatorname{Cos} c + 1} .$$

$$\Sigma a^x \operatorname{Sin}(b+cx)$$

$$= \frac{a}{a^2-2a \operatorname{Cos} c+1} . \frac{a^x (a \operatorname{Sin}(b-2c+cx) - \operatorname{Sin}(b-c+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$- \frac{1}{a^2-2a \operatorname{Cos} c+1} . \frac{a^x (a \operatorname{Sin}(b-c+cx) - \operatorname{Sin}(b+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$= \frac{a^x (a^2 \operatorname{Sin}(b-2c+cx) - 2a \operatorname{Sin}(b-c+cx) + \operatorname{Sin}(b+cx))}{(a^2-2a \operatorname{Cos} c+1)^2}$$

$$\Sigma^3 a^x \operatorname{Sin}(b+cx) = \Sigma \frac{a^x (a^2 \operatorname{Sin}(b-2c+cx) - 2a \operatorname{Sin}(b-c+cx) + \operatorname{Sin}(b+cx))}{(a^2-2a \operatorname{Cos} c+1)^2}$$

$$= \frac{a^2}{(a^2-2a \operatorname{Cos} c+1)^2} \Sigma a^x \operatorname{Sin}(b-2c+cx) - \frac{2a}{(a^2-2a \operatorname{Cos} c+1)^2} .$$

$$\Sigma a^x \operatorname{Sin}(b-c+cx) + \frac{1}{(a^2-2a \operatorname{Cos} c+1)^2} \Sigma a^x \operatorname{Sin}(b+cx)$$

$$= \frac{a^2}{(a^2-2a \operatorname{Cos} c+1)^2} . \frac{a^x (a \operatorname{Sin}(b-3c+cx) - \operatorname{Sin}(b-2c+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$- \frac{2a}{(a^2-2a \operatorname{Cos} c+1)^2} . \frac{a^x (a \operatorname{Sin}(b-2c+cx) - \operatorname{Sin}(b-c+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$+ \frac{1}{(a^2-2a \operatorname{Cos} c+1)^2} . \frac{a^x (a \operatorname{Sin}(b-c+cx) - \operatorname{Sin}(b+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$= a^x$

$$= \left\{\frac{a^x\,(a^3 \operatorname{Sin}(b-3c+cx) - 3a^2 \operatorname{Sin}(b-2c+cx) + 3a \operatorname{Sin}(b-c+cx) - \operatorname{Sin}(b+cx))}{(a^2-2a \operatorname{Cos} c+1)^3}\right\}$$

$$\Sigma^4 a^x \operatorname{Sin}(b+cx) = \left\{\frac{a^x\,(a^3 \operatorname{Sin}(b-3c+cx) - 3a^2 \operatorname{Sin}(b-2c+cx) + 3a \operatorname{Sin}(l-c+cx) - \operatorname{Sin}(b+cx))}{(a^2-2a \operatorname{Cos} c+1)^3}\right\}$$

$$= \frac{a^3}{(a^2-2a \operatorname{Cos} c+1)^3} \Sigma a^x \operatorname{Sin}(b-3c+cx) - \frac{3a^2}{(a^2-2a \operatorname{Cos} c+1)^3}.$$

$$\Sigma a^x \operatorname{Sin}(b-2c+cx) + \frac{3a}{(a^2-2a \operatorname{Cos} c+1)^3} \Sigma a^x \operatorname{Sin}(b-c+cx)$$

$$- \frac{1}{(a^2-2a \operatorname{Cos} c+1)^3} \Sigma a^x \operatorname{Sin}(b+cx)$$

$$= \frac{a^3}{(a^2-2a \operatorname{Cos} c+1)^3} \cdot \frac{a^x\,(a \operatorname{Sin}(b-4c+cx) - \operatorname{Sin}(b-3c+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$- \frac{3a^2}{(a^2-2a \operatorname{Cos} c+1)^3} \cdot \frac{a^x(a \operatorname{Sin}(b-3c+cx) - \operatorname{Sin}(b-2c+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$+ \frac{3a}{(a^2-2a \operatorname{Cos} c+1)^3} \cdot \frac{a^x\,(a \operatorname{Sin}(b-2c+cx) - \operatorname{Sin}(b-c+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$- \frac{1}{(a^2-2a \operatorname{Cos} c+1)^3} \cdot \frac{a^x\,(a \operatorname{Sin}(b-c+cx) - \operatorname{Sin}(b+cx))}{a^2-2a \operatorname{Cos} c+1}$$

$$= \left\{\frac{a^x(a^4 \operatorname{Sin}(b-4c+cx) - 4a^3 \operatorname{Sin}(b-3c+cx) + 6a^2 \operatorname{Sin}(b-2c+cx) - 4a \operatorname{Sin}(b-c+cx) + \operatorname{Sin}(b+cx))}{(a^2-2a \operatorname{Cos} c+1)^4}\right\},$$

. . . .

sicque

sicque porro; unde per inductionem in genere concluditur

$$\Sigma^n{}_a{}^x \mathrm{Sin}(b+cx) = \left\{ \frac{\begin{array}{l} a^x \left(a^n \mathrm{Sin}\,(b-nc+cx) - na^{n-1} \mathrm{Sin}\,(b-(n-1)\,c+cx) \right. \\ + \frac{n(n-1)}{1.2} a^{n-2} \mathrm{Sin}\,(b-(n-2)c+cx) - \frac{n(n-1)(n-2)}{1.2.3} a^{n-3} \\ \left. \mathrm{Sin}\,(b-(n-3)\,c+cx) + \&c. \right) \end{array}}{(a^2 - 2a\,\mathrm{Cos}\,c + 1)^n} \right\},$$

continuata scilicet serie numeratoris, usque quo termini ejus sponte evanescant.

Positis hac in æquatione

$$b = 1^q,\ c = o,$$

mutabitur ea in

$$\Sigma^n a^x = \frac{a^x \left(a^n - na^{n-1} + \frac{n(n-1)}{1.2} a^{n-2} - \&c.\right)}{(a^2 - 2a + 1)^n}$$

$$= \frac{a^x (a-1)^n}{(a-1)^{2n}} = \frac{a^x}{(a-1)^n};$$

unde series igitur habetur valorum particularium

$$\Sigma a^x = \frac{a^x}{a-1}$$

$$\Sigma^2 a^x = \frac{a^x}{(a-1)^2}$$

$$\Sigma^3 a^x = \frac{a^x}{(a-1)^3}$$

$$\Sigma^4 a^x = \frac{a^x}{(a-1)^4}$$

. . . . .

quas applicandi occasio saepe satis se offert.

Posito vero $a = 1$, abibit allata nuper formula in

$$\Sigma^n \mathrm{Sin}(b+cx) = \left\{\frac{\mathrm{Sin}\,(b-nc+cx) - n\,\mathrm{Sin}\,(b-(n-1)c+cx) + \frac{n(n-1)}{1\,.\,2}\,\mathrm{Sin}\,(b-(n-2)c+cx) - \&c.}{(2-2\,\mathrm{Cos}\,c)^n}\right\}$$

$$= \left\{\frac{\mathrm{Sin}\,(b-nc+cx) - n\,\mathrm{Sin}\,(b-nc+cx+c) + \frac{n(n-1)}{1\,.\,2}\,\mathrm{Sin}\,(b-nc+cx+2c) - \&c.}{2^{2n}\,\mathrm{Sin}\,\frac{1}{2}\,c^{2n}}\right\};$$

unde sequitur quidem, si n numerus sit *par*, haberi